Comment on "Lateral Casimir force beyond the proximity-force approximation"

F. Chen[1], U. Mohideen[1], G. L. Klimchitskaya[2], and V. M. Mostepanenko[2]
[1]*Department of Physics and Astronomy, University of California, Riverside, California 92521, USA*
[2]*Institute for Theoretical Physics, Leipzig University, Augustusplatz 10/11, 04109 Leipzig, Germany*

PACS numbers: 12.20.Ds, 11.10.Wx, 12.20.Fv

The Letter [1] is devoted to the calculation of the lateral Casimir force arising between the corrugated metallic bodies (two parallel plates or a sphere above a plate at a separation L) taking corrugations into account without use of the proximity force approximation (PFA). The metal of the test bodies is described by the plasma model (ω_P is the plasma frequency), the corrugation wavelength is λ_C and the amplitudes are $a = a_1, a_2$. The lateral Casimir force with the amplitude of order of a_1a_2 is found using the scattering approach. As the authors themselves note, "This technique is valid as long as... $a \ll L, \lambda_C, \lambda_P$." Previously the lateral Casimir force was measured in Ref. [2]. It was also calculated for test bodies made of ideal metal outside the PFA [3] and of real metals described by the plasma model using the PFA [2]. The main result of Ref. [1] is that "in realistic experimental situations the proximity force approximation overestimates the force up to 30%." Below we demonstrate that the approach in Ref. [1] is in disagreement with a path-integral theory of Ref. [3], and the approximations used are not appropriate for comparison to experiment and for making statements on the accuracy of the PFA.

We start with the computational results in Fig.1 of [1] for the variation of function ρ versus $k = 2\pi/\lambda_C$ describing the deviation from PFA in the case of two parallel plates. According to this figure, with $\lambda_C = 1.2\mu$m at $L = 200$nm the lateral force amplitude is less by 16% than the value given by the PFA. This is, however, in contradiction with a more fundamental theory [3] formulated for ideal metals. It is easily seen, that the quantity ρ, plotted in Fig.1 as a function of k at different L, is, in fact, a function of kL. Thus, according to the Letter, for corrugated plates with the rescaled $\lambda_C = 12\mu$m and $L = 2\mu$m the deviation of the lateral force amplitude from the PFA value is still 16%. At $L = 2\mu$m, however, the role of nonideality of a metal is very small, and Fig.5 of Ref. [3] demonstrates the complete agreement between the exact result and the PFA if (as it holds in our case) L is several times less than λ_C. If this condition is not satisfied, the PFA underestimates the force amplitude [3], and not overestimates it as in [1]. In the nonperturbative regime the same is demonstrated in Ref. [4].

For the experimental configuration of a sphere at a separation $L = 221$ nm above a plate Ref. [1] obtains the "exact" computational value of 0.20 pN for the amplitude of the lateral force with all experimental parameters as in Ref. [2] ($\lambda_C = 1.2\mu$m, $\lambda_P = 136$ nm, $a_1 = 59$ nm, and $a_2 = 8$ nm). According to Ref. [1], the linear in a_1a_2 version of the PFA gives instead 0.28 pN, i.e., 40% difference. However, Ref. [1] does not claim that the PFA overestimates the force by 40%, but employs the following reasoning: "Precisely, Ref. [2] finds a force of 0.32 pN at $L = 221$ nm, with a relative correction due to higher powers of 1.21. Discounting this factor, the second-order force should be 0.26 pN, which overestimates the exact result by a factor of the order of 30%."

At this point we stress that in Ref. [2] the force amplitude at $L = 221$ nm was both measured and computed using the complete PFA accounting for all powers in a_1, a_2. The measured and theoretical values of 0.32±0.077 pN (at 95% confidence) and 0.33 pN, respectively, are in a very good agreement. It is illogical that Ref. [1] "discounts" the effect due to higher powers in a_1, a_2 in our result if it aims to propose some general statements on the accuracy of the PFA. In our experiment a_1 and a_2 are not small compared to L (for instance, $a_1/L = 0.27$) and it is insufficient to restrict to the first power in a_1a_2 as is done in Ref. [1]. The calculations of Ref. [1] are also performed under the assumption that $a_1, a_2 \ll \lambda_P$ which is not met in our experiment because $a_1/\lambda_P = 0.43$. As a result, Ref. [1] arrives at a force amplitude of 0.20 pN so far away from the measured value of 0.32 pN and theoretical value of 0.33 pN using the complete PFA [2]. The force amplitude of 0.20 pN found in the Letter, when compared to experiment, could be realized only with the probability of 0.06%. In the approach of Ref. [1] the force amplitude is underestimated by 30–40% compared to both the PFA prediction and experiment. This approach is inadequate, because the equations, obtained to the lowest orders in small parameters, were applied outside of their application range, and the results obtained are not in agreement with the path-integral theory of Ref. [2]. Notice that the linear version of the PFA [1] leads to 0.28 pN force amplitude instead of 0.20 pN, i.e., much closer to the measured and computed using the complete PFA [2] values of 0.32 pN and 0.33 pN, respectively. This is in agreement with the result of Refs. [3, 4] obtained for ideal metal plates that the PFA works well when, as it holds in our experiment, the separation L is several times less than λ_C.

[1] R. B. Rodrigues, P. A. Maia Neto, A. Lambrecht, and S. Reynaud, Phys. Rev. Lett. **96**, 100402 (2006).

[2] F. Chen, U. Mohideen, G. L. Klimchitskaya, and V M. Mostepanenko, Phys. Rev. Lett. **88**, 101801 (2002); Phys. Rev. A **66**, 032113 (2002).

[3] T. Emig, A. Hanke, R. Golestanian, and M. Kardar, Phys. Rev. A **67**, 022114 (2003).

[4] R. Büscher and T. Emig, Phys. Rev. Lett. **94**, 133901 (2005).